Modeling Wave Dispersion in Bound States in the Continuum (BIC) Media

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Abstract

Bound states in the continuum (BIC) are waves that remain localized even though they coexist with a continuous spectrum of radiating waves that can carry energy away. This behaviour is different from conventional knowledge. It was first proposed in Quantum Mechanics. BIC are a general wave phenomena identified in electromagnetic, water, acoustic and elastic waves. The unique properties of BIC gave rise to wide range of applications from lasers and sensors to filters and low-loss fibers. An effect of BIC can be dispersion of a wave when passed through such a medium. In this document, we try to model Dispersion of a signal when passed through a medium.

1 Introduction

There are many studies on BIC in variety of material systems; piezoelectric, dielectrics, optical waveguides and fibres and photonic crystals to name a few. The goal is to understand BIC role in dispersion of a signal(and study temporal response in a passive BIC cavity?).

Temporal dispersion of a signal when passed through a dispersive medium can be traced back to group velocity dispersion (GVD), which is a characteristic of dispersive medium. In simple terms, group velocity is the speed at which a pulse(modulated or un-modulated) propagates; this means the wave envelope travels at a different speed than of the constituting waves. GVD is often used to determine how a dispersive medium will affect a signal. Depending on the parameters, it could be responsible for dispersive temporal broadening or compression of ultrashort pulses. GVD is given as derivative of inverse of group velocity, v_g w.r.t angular frequency, ω as follows:

$$\text{GVD} = \frac{\partial}{\partial \omega}\frac{1}{v_g} = \frac{\partial^2 k}{\partial \omega^2}, \tag{1}$$

where, k = frequency-dependent wavenumber.

To progress in the direction of understanding temporal dispersion, we first understand the mathematics of dispersion of signal and importantly how one can model the interaction of signal and a medium. For this purpose, we go back to the fundamentals of signal processing. Any medium by its transfer function, also know as impulse response. When a signal, say $x(t)$, passes through a medium with impulse response $h(t)$ output signal is given as:

$$y(t) = x(t) * h(t), \tag{2}$$

where, * is the convolution operator.

Aliter, Fourier transform of the signal and impulse response can be used. Let, $X(f)$ and $H(f)$ are Fourier transforms of $x(t)$ and $h(t)$ respectively. The output is inverse Fourier transform of product of both the signals, i.e.,

$$y(t) = \mathcal{F}^{-1}(X(f)H(f)) \tag{3}$$

The system response can be anything — in the simplest case, it can be a Lorentz oscillator. In this simulation, we used Lorentzian (refer section A.3) as the system response, to mimic a dispersive medium. We implemented the above functions and transforms in `Python3` using `numpy` library and `matplotlib` library for visualisation. (refer A.1 and A.2)

Link for the code.

2 Results and Discussion

For understanding dispersion through a medium with impulse response, Lorentzian (section A.3), χ, we studied it on various signals; Gaussian, exponential, sinusoidal and enveloped signals.

The equation used to obtain output from the dispersed medium is as follows:

$$y(t) = \mathcal{F}^{-1}(X(f)\epsilon(f)\chi(f)), \tag{4}$$

where, ϵ is the permittivity of the medium and χ is the transfer function of the medium.

To achieve dispersion of a signal, first, we apply Fourier transform to the given signal. Next, a Lorentzian is generated(in frequency domain) such that the peak is aligned with the central frequency of the transformed signal. This Lorentzian in principal acts like an amplitude modulated impulse train, shifting/scaling the signal in frequency domain, hence, stretching the signal.

We varied the input signal type, simultaneously modifying the Lorentzian such that the peaks align and giving rise to a stretched pulse. The plots are normalised for visualisation. A single plot, Fig. 3, showing no normalisation is placed to show the energy is conserved; the amplitude decreases to compensate for stretching, hence keeping the area under the curve constant.

In the following sections, from section A.3, Γ is the full width at half maximum (FWHM) of the Lorentzian until mentioned otherwise.

2.1 Gaussian pulse

A Gaussian pulse centred at μ and standard deviation σ is generated using the following equation:

$$x(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-\mu)^2}{\sigma^2}}. \tag{5}$$

The relation between $\sigma$and FWHM is given as:

$$\sigma = \frac{\text{FWHM}}{2\ln 2}, \tag{6}$$

this relation is used to calculate σ from a given FWHM of the pulse.
By substituting Eq. 5 in Eq. 4, with varying parameters, we observe that with increasing Γ, the dispersion decreases as evident from Figs. 1 and 2.

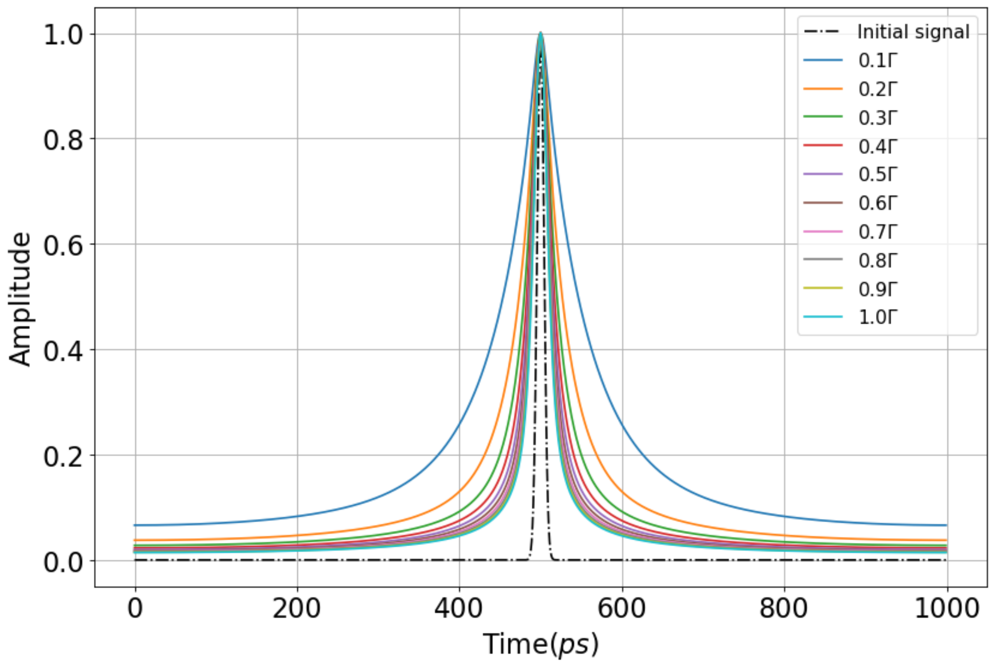

Figure 1: FWHM = 10 ps, varying the FWHM of the Lorentzian, Γ = 100 GHz.

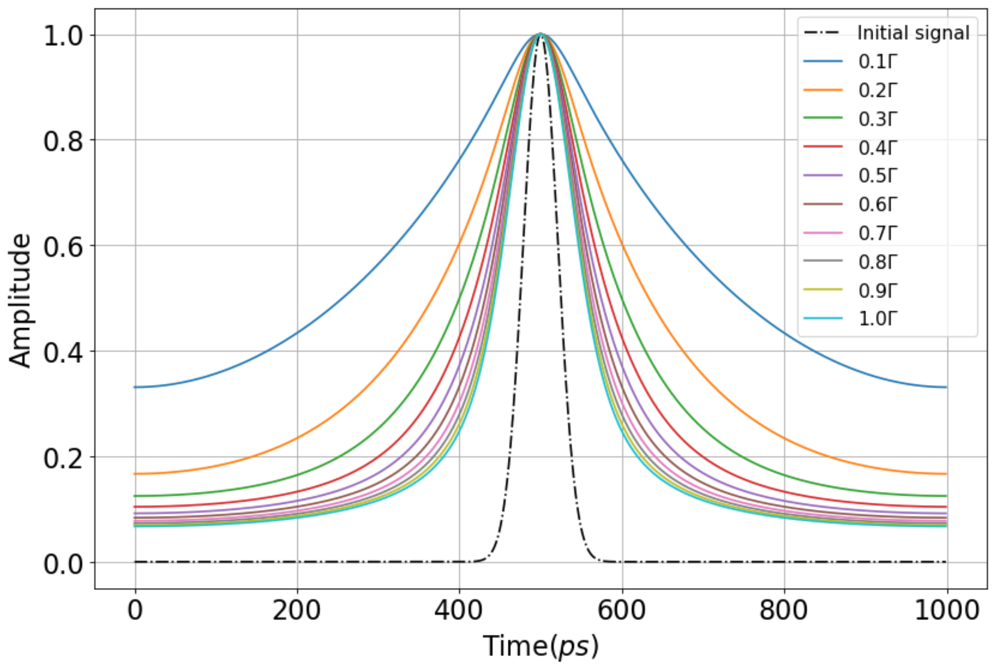

Figure 2: FWHM = 50 ps, varying the FWHM of the Lorentzian. Γ = 20 GHz.

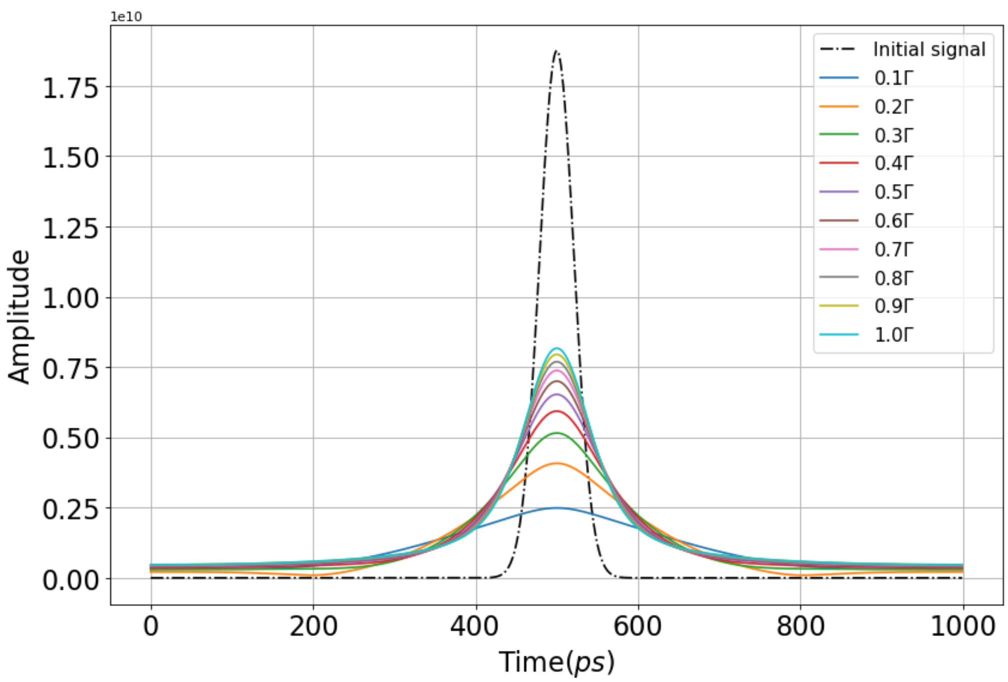

Figure 3: Non-normalised: FWHM = 50 ps, varying the FWHM of the Lorentzian, Γ = 20 GHz.

2.2 Exponential pulse

- **Exponential decay only**: An exponential pulse with the factor τ is generated using the following equation:

$$x(t) = A_0 e^{-t/\tau}, \tag{7}$$

  where, A_0 is the max amplitude of the function. The result for an exponentially decaying signal is shown in Fig. 4. We observe that the dispersion increases with decreasing Γ of the Lorentzian.

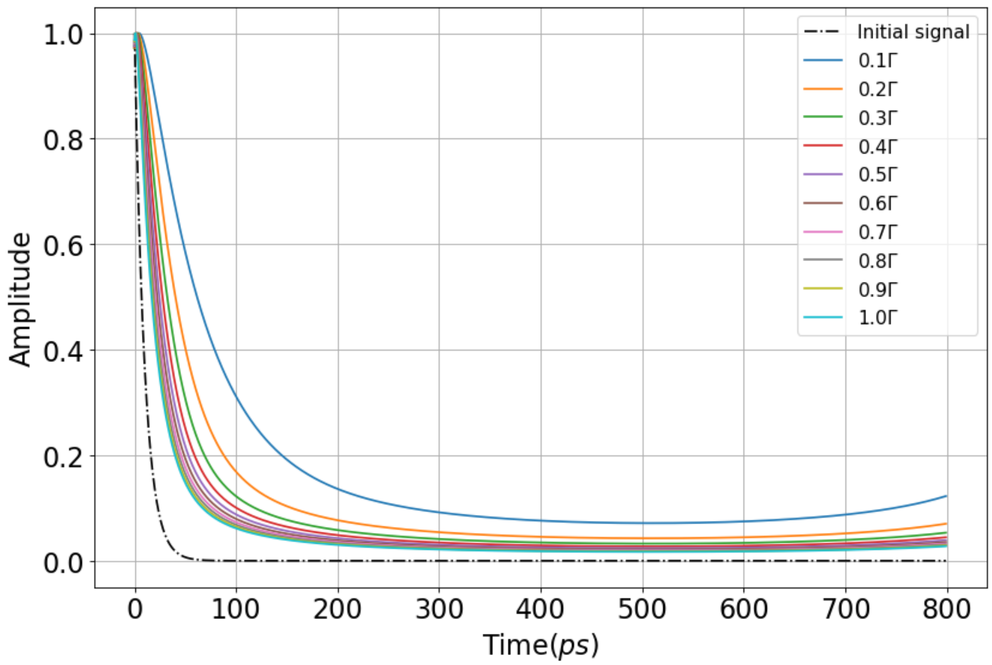

Figure 4: $\tau = 10$ ps, varying the FWHM of the Lorentzian, $\Gamma = 100$ GHz.

- **Exponential rise and fall:** An exponentially rising and falling pulse with the factor $\tau = 10$ ps is generated as shown in Fig. 5.

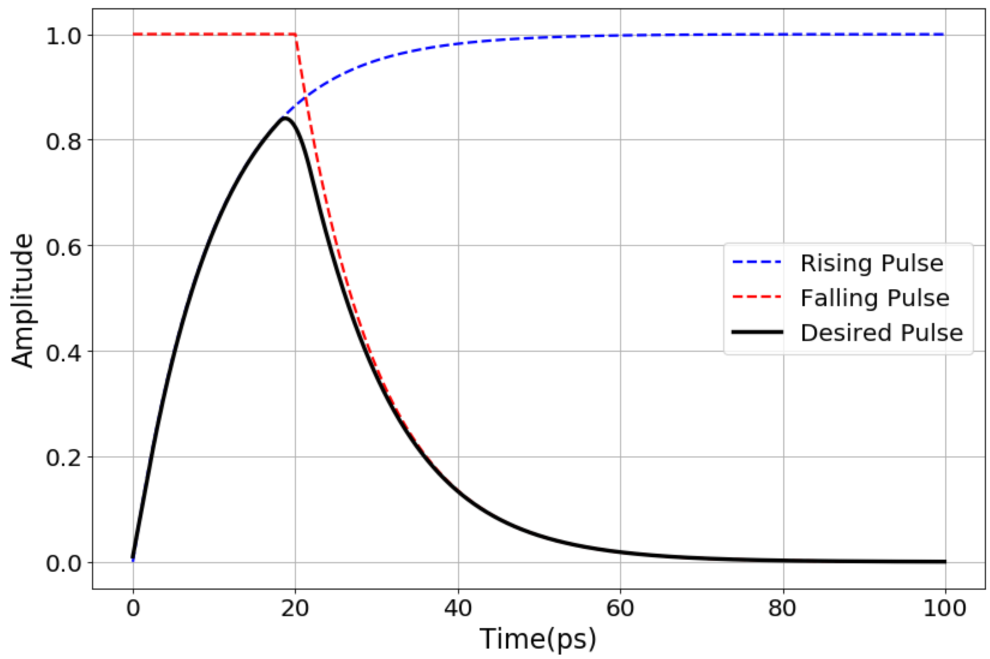

Figure 5: $\tau = 10$ ps

The dispersed signal obtained with the FWHM of the Lorentzian, $\Gamma = 100$ GHz is in Fig. 8

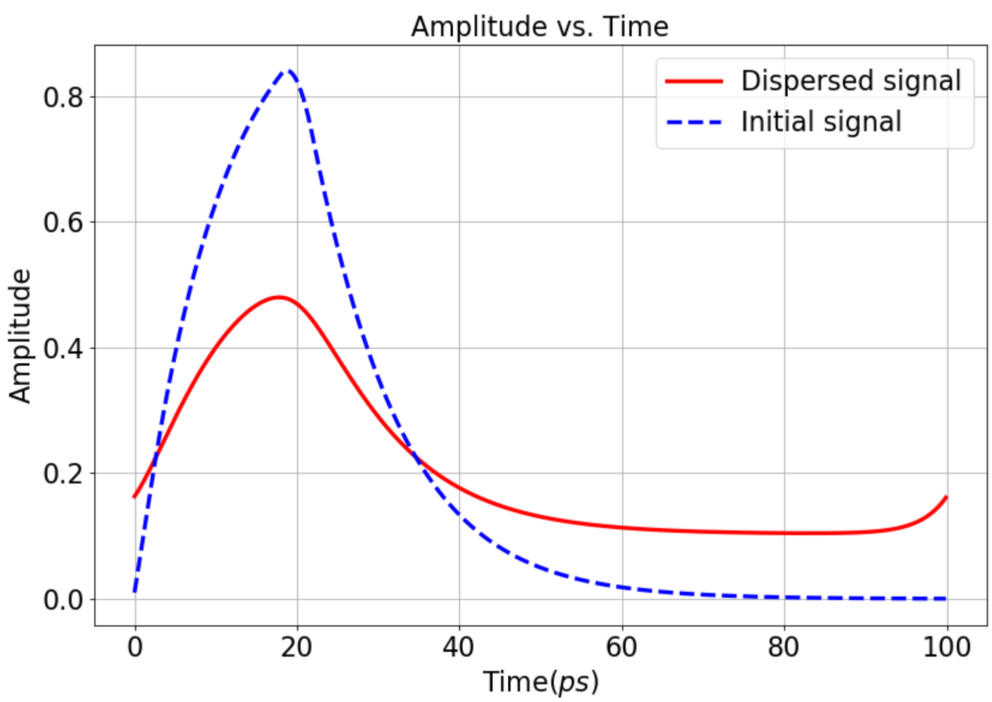

Figure 6: Non-normalised: $\tau = 10$ ps, $\Gamma = 100$ GHz.

2.3 Sinusoidal Signal

Similarly, for a sinusoidal signal, here frequency = 375 GHz, we obtain the following results

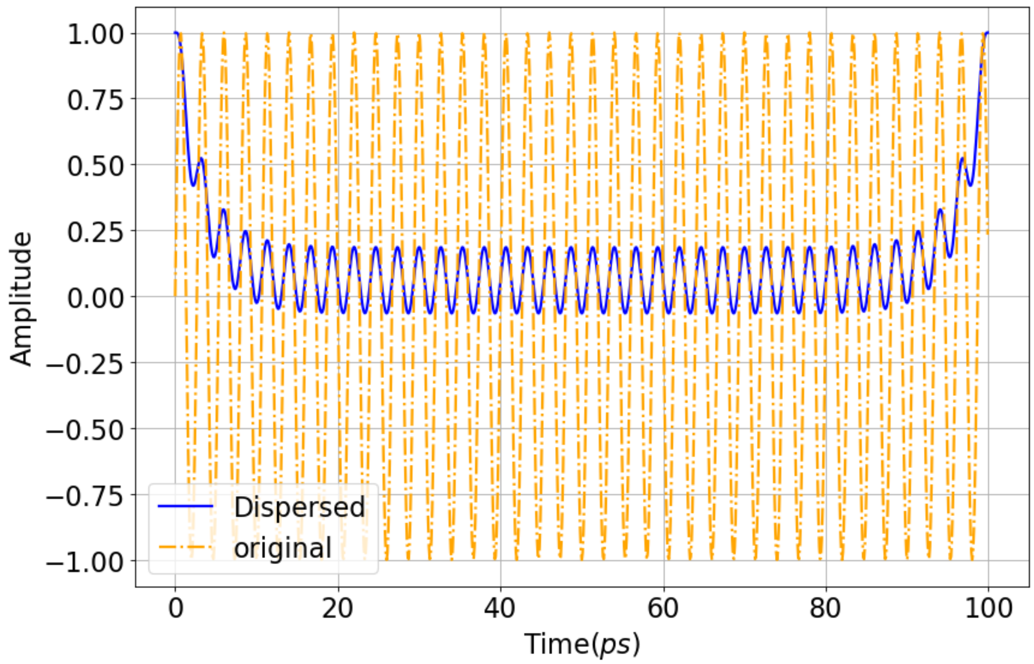

Figure 7: Non-normalised: Frequency = 375 GHz, $\Gamma = 375$ GHz

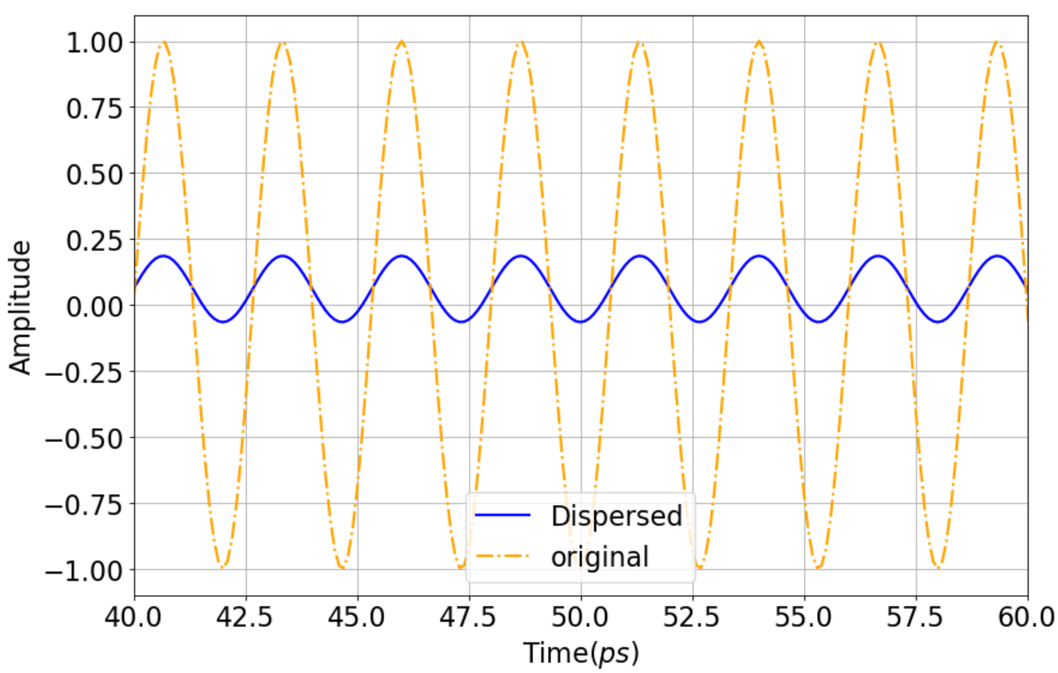

Figure 8: Non-normalised, zoomed graph: Frequency = 375 GHz, $\Gamma = 10$ GHz.

2.4 Modulated Signal

A signal is amplitude modulated with a sine wave creating an envelope. More about generation of the signal can be found in Appendix A.4 The results are as in Figs. 9 and 10.

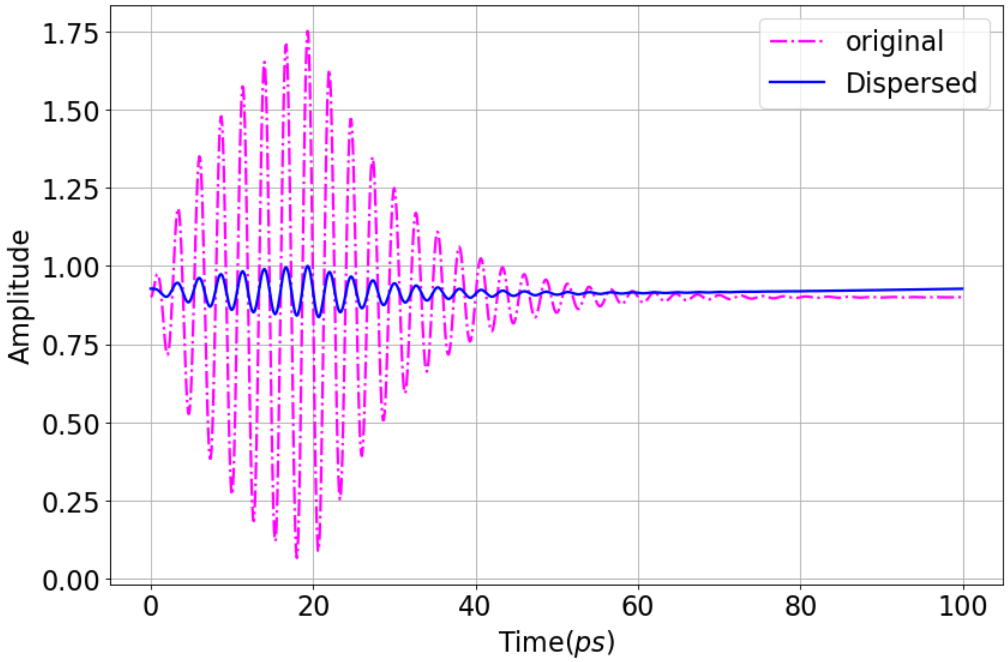

Figure 9: Frequency of sinusoid = 375 GHz, exponential τ = 10 ps and FWHM of Lorentzian = 50 GHz.

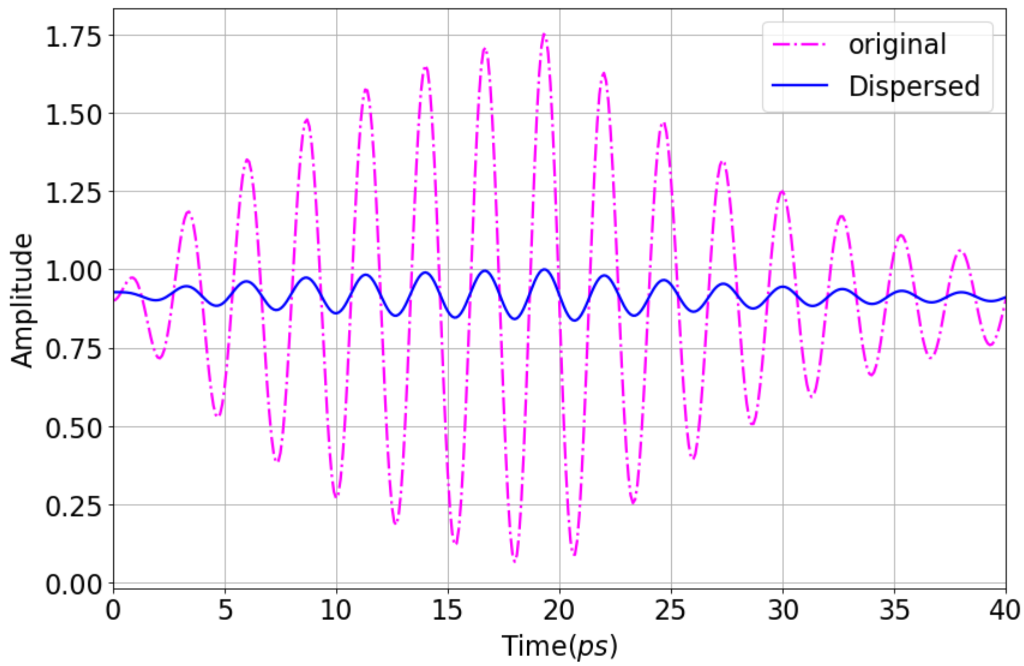

Figure 10: Zoomed: Frequency of Sinusoid = 375GHz, Exponential $\tau = 10ps$ and FWHM of Lorentzian $= 50GHz$

3 Conclusion

A Lorentzian acts a band-pass filter allowing only certain frequencies to pass, this behaviour is directly reflected in changing of the centre and FWHM of the Lorentzian directly affecting the output signal. This very concept of band-pass filters is exploited in making various optical devices and lenses. These filters are obtained by stacking layers of dielectrics or any suitable material using fabrication techniques. The simplest and everyday example would be lenses in glasses; it coated with particular substances to reflect blue light or UV light as desired by the user.

4 Stack Simulation

4.1 Parameters

$n_{SUP} = 1$
$n_H = 2.6$
$n_L = 1.34$
$n_{SUB} = 1.5$
No of layers = 18
Thickness of each layer = 79 nm

4.2 Plots

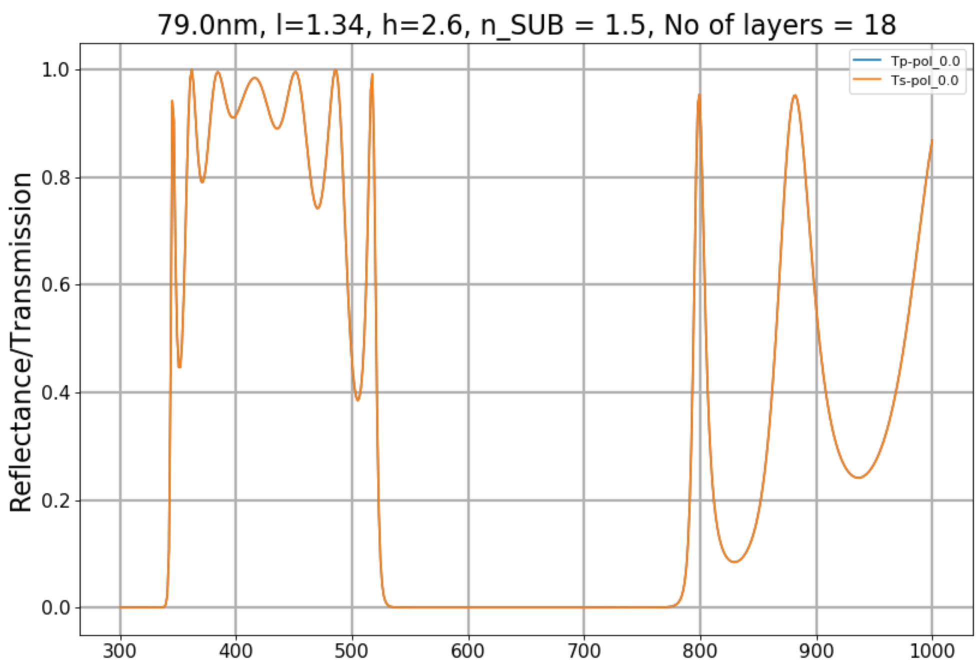

Figure 11: Data from Tmatrix Python script. The title gives the values of all parameters

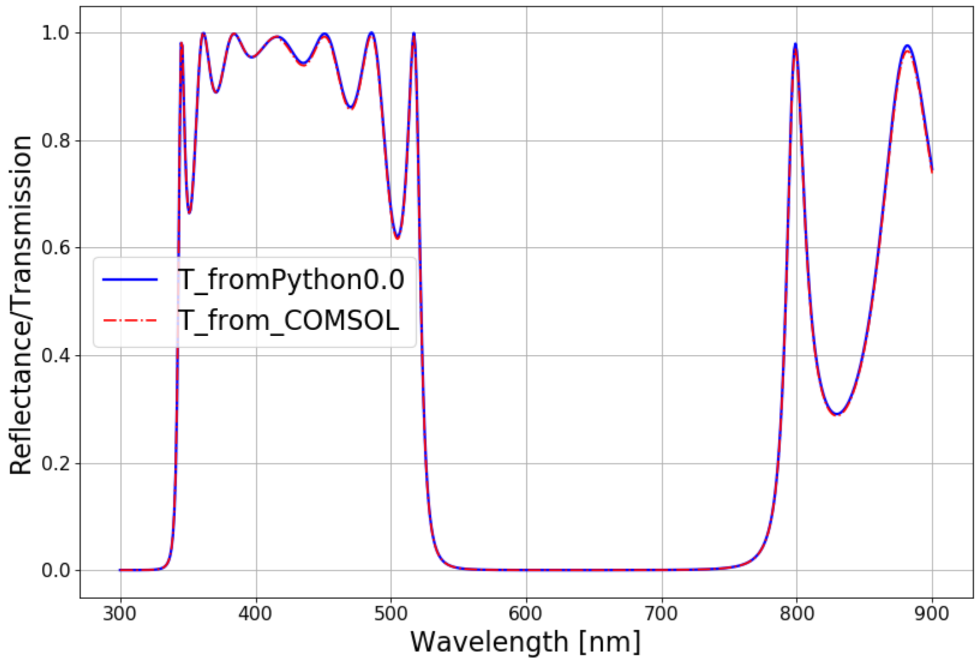

Figure 12: Data from Tmatrix Python script compared with COMSOL Simulation

5 Lorentz Oscillator

The following equation is taken from COMSOL equation part.

$$\epsilon_r = \epsilon_{\text{inf}} + \frac{strength}{{w_0}^2 - w^2 + j\Gamma w} \tag{8}$$

here, $w_0 =$ Centre of the Lorentzian
$\Gamma =$ Width of the Lorentzian
$\eta = \sqrt{\epsilon_r}$

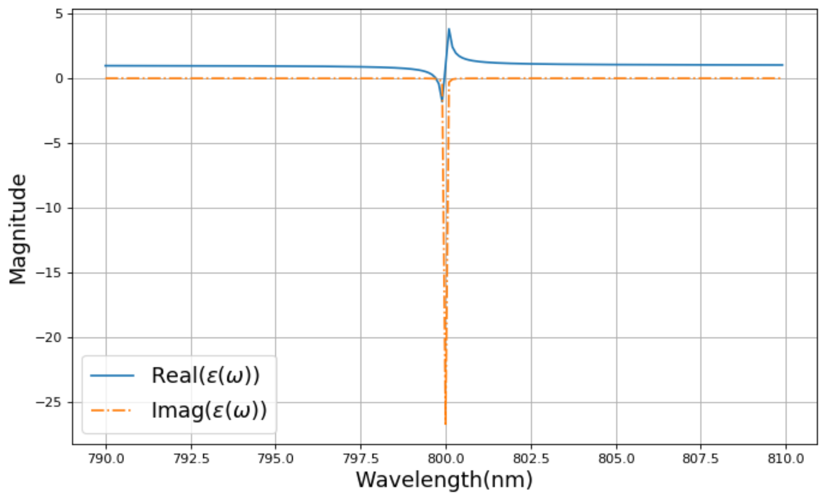

Figure 13: Relative Permittivity from Lorentz Oscillator

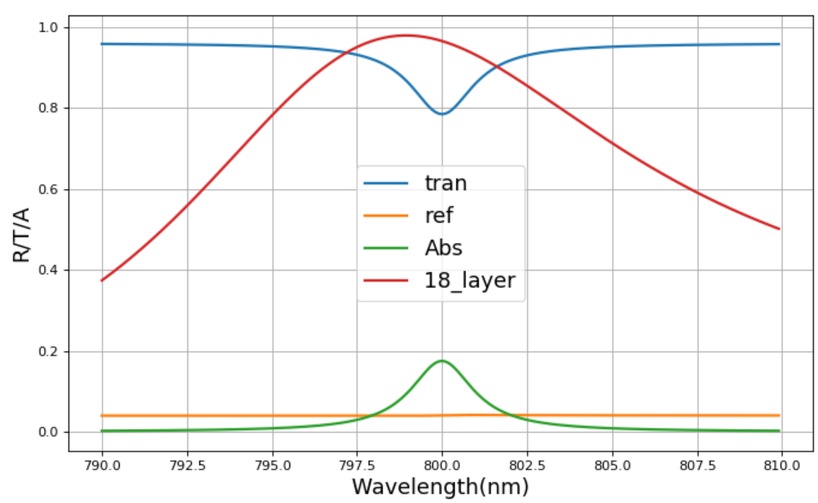

Figure 14: The red curve is the Transmittance of Stack from COMSOL. The other curves are Reflection,Transmission and Absorption from a single layer(thickness=18*79nm) with effective permittivity.

There is a strong dispersion at 800nm wavelength, and we observe absorption at 800nm, adjusting the strength and Gamma gives a strong absorption.**The expected result is transmission around 800nm.**

A Appendix

A.1 Sampling

In the implementation, we sampled any signal as follows: Let $x(t)$ be any signal, where $x(t) < \infty$ and T be the sampling time period

$$\implies \text{Sampling frequency}, F_s = \frac{1}{T}$$

The sampling is as follows

$$x[n] = x(t)_{t=nT}, \text{ where } n = 0, 1, \ldots, N$$

Let $X(j\Omega)$, where $\Omega = 2\pi f$ be the Fourier transform of $x(t)$. Similar to time domain, sampling is as follows

$$X[k] = X(j\Omega)_{\Omega=\frac{2\pi k}{NT}}, \text{ where } k = 0, 1, \ldots, N$$

Here, F_s satisfies the Nyquist criteria and a signal is analysed in the window $(0, NT)$.

A.2 Discrete Fourier Transform

Given a sequence of N complex numbers $\mathbf{x_n} := x_0, x_1, \ldots, x_{N-1}$, the application of Fourier transform gives a set of N complex numbers $\mathbf{X_n} := X_0, X_1, \ldots, X_{N-1}$.

The relation between $\mathbf{x_n}$ and $\mathbf{X_n}$ is given by

$$X_k = \sum_{n=0}^{N-1} x_n \exp -j\frac{2\pi kn}{N} \tag{9}$$

and

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k \exp j\frac{2\pi nk}{N} \tag{10}$$

Equation 9 is Fourier transform and equation 10 is inverse Fourier transform.

A.3 Lorentzian

For the simulations, we have sampled the signals at a suitable sampling frequency, F_s with N points in the sequence. The Lorentzian function is given by

$$\chi(x) = \frac{1}{\pi} \frac{\frac{\Gamma}{2}}{(x - x_0)^2 + (\frac{\Gamma}{2})^2}$$

The factor $\frac{1}{\pi}$ is such that

$$\int_{-\infty}^{\infty} \chi(x) dx = 1$$

The function is defined by it's FWHM, Γ and centred around x_0. Figure 15 shows a Lorentzian with $\Gamma = 100THz$ and centred at $1875THz$

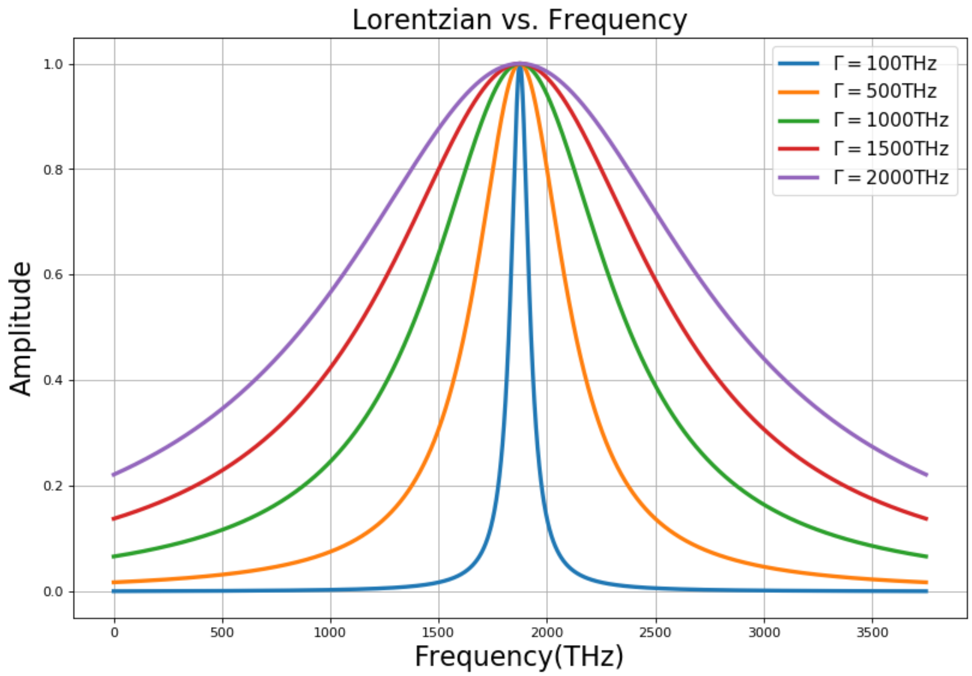

Figure 15: Lorentzian centred at $= 1875THz$, varying FWHM

A.4 Amplitude Modulation

In amplitude modulation (AM), a given signal's amplitude is modulated in time. This is usually achieved by multiplying with a carrier signal, a sinusoid with high frequency. For example, in figure 16, a Gaussian pulse is multiplied with a sine function of desired wavelength to get the Input Signal.

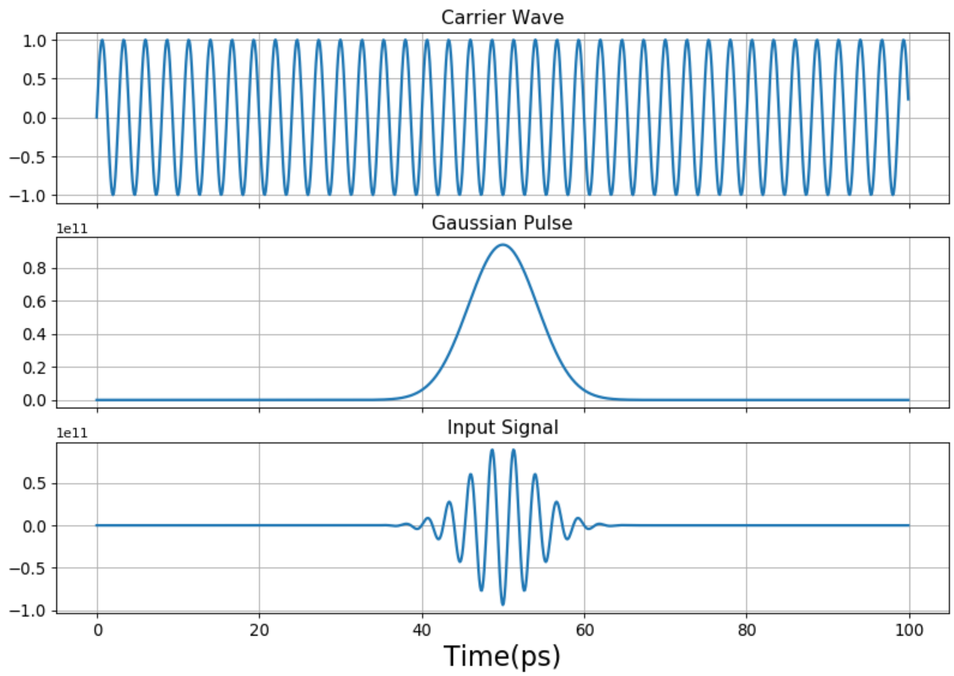

Figure 16: (a) Sinusoid Frequency = 375GHz (b) Gaussian FWHM = 10ps (c) Product of Carrier wave and Gaussian Pulse

References

[1] https://www.rp-photonics.com/group_velocity_dispersion.html

[2] http://www.mit.edu/~soljacic/BIC_review-NatRevMat.pdf

[3] https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-007-electromagnetic-energy-from-motors-to-lasers-spring-2011/readings/MIT6_007S11_lorentz.pdf

[4] https://archive.lib.msu.edu/crcmath/math/math/l/l417.html